Enrollment No: ______________________ Exam Seat No: ______________________

C.U.SHAH UNIVERSITY

Summer Examination-2017

Subject Name: Problem Solving-III

Subject Code: 5SC04PRS1 **Branch: M.Sc. (Mathematics)**

Semester: 4 **Date: 20/04/2017** **Time: 10:30 To 01:30** **Marks: 70**

Instructions:

(1) Use of Programmable calculator and any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

SECTION – I

Q-1 Answer the Following questions: **(07)**

a. Q and Q^c are measurable. – True or False? **(01)**

b. Define: Reimann integrable function. **(02)**

c. Define monotonic sequence with an example. **(02)**

d. Define: L(P,f) and U(P,f) for any bounded function f on [a, b]. **(02)**

Q-2 Attempt all questions **(14)**

a. Find the measure of *E*, if *E* is a singleton subset of R. **(02)**

b. Prove that $\lim_{n\to\infty}\frac{1}{n}[\,1+\frac{1}{2}+\frac{1}{3}+\ldots\ldots\frac{1}{n}\,]=0$. **(04)**

c. Find lim(sup) and lim(inf) for the following. **(08)**

i) $\{(-1)^n\}$ ii) $\{\,(1+\frac{1}{n})^n\,\}$ iii) $\{\sin\frac{n\pi}{2}\,\}$ iv) $\{\,1,\frac{1}{2},\frac{1}{3},\frac{1}{4},\ldots\ldots\ldots,0\,\}$

OR

Q-2 Attempt all questions **(14)**

a. For which value of *p*, the series $\frac{1}{1^p}+\frac{1}{2^p}+\frac{1}{3^p}+\ldots.+\frac{1}{n^p}+\ldots.$ is convergent? **(01)**

b. Test the convergence of the series $(\frac{2^2}{1^2}-\frac{2}{1})^{-1}+(\frac{3^3}{2^3}-\frac{3}{2})^{-2}+(\frac{4^4}{3^4}-\frac{4}{3})^{-3}+\ldots.$ **(04)**

c. Test the convergence for the following series. **(09)**

i) $\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}+\sqrt{n+1}}$ ii) $\frac{1}{4.7.10}+\frac{1}{7.10.13}+\frac{1}{10.13.16}+\ldots\ldots$ iii) $\sum_{n=1}^{\infty}\frac{1}{n\left(1+\log^2 n\right)}$

Q-3 Attempt all questions **(14)**

a. Find L(P, f) and U(P,f) for f(x)= 2x on [0,2]. **(04)**

b. If E_1 and E_2 are measurable set then prove that $m(E_1 \cup E_2)+m(E_1 \cap E_2)=m(E_1)+m(E_2)$. **(04)**

c. Show that x^2 is integrable on [0,1]. **(06)**

OR

Q-3 Attempt all questions **(14)**

a. i) State Leibnitz test for alternating series. **(04)**

ii) If f(x) =[x] on [1,2] then find the value of $\int_1^2 f(x)$dx.

b. Show that $\lim_{n\to\infty} \left[\left(\frac{2}{1}\right)\left(\frac{3}{2}\right)^2\left(\frac{4}{3}\right)^3 \ldots\ldots \left(\frac{n+1}{n}\right)^n\right]^{\frac{1}{n}} = e$. **(04)**

c. Check that which of the following series are absolutely convergent. **(06)**

i) $1-\frac{1}{2.3}+\frac{1}{2.3.4}-\frac{1}{2.3.4.5}+\ldots\ldots$ ii) $\sum_{n=1}^{\infty} (-1)^n \frac{1}{n^{5/3}}$.

SECTION - II

Q-4 Answer the Following questions: **(07)**

a. How many positive integers which are less than 108 and prime to 108? **(01)**

b. Let $X=\{a,b,c\}$ and $\tau=\{\phi,\{a\},\{b\},\{a,b\},X\}$ then find the interior of **(02)**

i) $A=\{b,c\}$ ii) $B=\{a,c\}$

c. How many numbers between 1000 and 10,000 can be formed with the digits 1,3,5,7,9 each digit being used only once in each number? **(02)**

d. Show that the topological space $(R,\cup)$ is first countable. **(02)**

Q-5 Attempt all questions **(14)**

a. Let $f: X \to Y$ be a function from a non-empty set X into a topological space (Y,u). Moreover let τ be the class of inverse of open subsets of Y; $\tau=\{f^{-1}[G]; G\in u\}$ then show that τ is a topology on X. **(04)**

b. Let A be a subset of a topological space (X,τ) then show that $\tau=\{A\cap G; G\in\tau\}$ is topology on A. **(04)**

c. In English language consists of 21 consonants and 5 vowels. How many 5 letter words, consisting of at least a vowel and two consonants can be formed from them? **(03)**

d. Consider the following topology on $X=\{1,2,3,4,5\}$ and let $\tau=\{\phi,\{1\},\{1,2\},\{1,2,5\},\{1,3,4\},\{1,2,3,4\},X\}$. List the neighborhood of point 1 and 3. **(03)**

OR

Q-5 Attempt all questions **(14)**

a. Consider the topology $\tau = \{\phi, \{a\}, \{c,d\}, \{a,c,d\}, \{b,c,d,e\} X\}$ on the set $X = \{a,b,c,d,e\}$ and show that **(04)**

i) (X, τ) is disconnected.

ii) $Y = \{b,d,e\}$ is a connected subset of *X*.

b. Consider the topology $\tau = \{\phi, \{a\}, \{a,b\}, \{a,c,d\}, \{a,b,c,d\}, \{a,b,e\}, X\}$ on $X = \{a,b,c,d,e\}$. Determined the derived sets of i) $A = \{c,d,e\}$, ii) $B = \{b\}$ **(04)**

c. How many permutations can be made using letters of the word mathematics? In how many of them vowels occurring together? **(03)**

d. Let τ be the class of subsets of *N* consisting ϕ and all subsets of *N* of the form $E_n = \{n, n+1, n+2, ...\}$ with $n \in N$. **(03)**

i) Determine the closed subset of (N, τ).

ii) Determine the closure of the sets $\{7, 24, 47, 85, ...\}$.

Q-6 Attempt all questions **(14)**

a. How many integers are between 1 and 200 which are divisible by any one of the integers 2,3 and 5? **(04)**

b. Let $X = \{1,2,3,4,5\}$ then find the topology τ on *X* generated by $B_* = \{\{a\}, \{c,d\}, \{a,b,c\}\}$. **(04)**

c. Find the total number of divisor of number 38808 excluding 1 and the number itself. **(03)**

d. Let $X = \{a,b,c,d,e\}$, determine that each of the following classes of subsets of *X* is a topology on *X* or not. **(03)**

i) $\tau_1 = \{\phi, \{a\}, \{a,b\}, \{a,c\}, X\}$

ii) $\tau_2 = \{\phi, \{a\}, \{a,b\}, \{a,c,d\}, \{a,b,c,d\}, X\}$

OR

Q-6 Attempt all Questions **(14)**

a. Solve $x \equiv 2 (\bmod 3)$, $x \equiv 3 (\bmod 4)$, $x \equiv 4 (\bmod 5)$ by Chinese remainder theorem. **(04)**

b. Find all integes *x* for which $5x \equiv 12 (\bmod 19)$. **(04)**

c. For any integer n, show that $(2n+1)$ and $(13n+6)$ are relatively prime. **(03)**

d. Consider the topology $\tau = \{\phi, \{1\}, \{3,4\}, \{1,3,4\}, \{2,3,4,5\}, X\}$ on $X = \{1,2,3,4,5\}$. List the members of the relative topology τ_A on $A = \{1,4,5\}$. **(03)**